Welcome! Probability

When asked to define a miracle–

"Anything with a probability of less than 10%"
-Enrico Fermi

Outline

Meet Linda[1]

Linda is 31 years old, single, outspoken, and very bright. She majored in philosphy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in the 2017 Women's March.

Assign proabilites for each statement that describes Linda:

1. Linda is a teacher.
2. Linda is a bookstore cashier and enjoys yoga.
3. Linda is a bank teller.
4. Linda describes herself as a feminist.
5. Linda sells insurance.
6. Linda is a bank teller and describes herself as a feminist.

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[1] Tversky & Kahneman experiment

Shared Birthdays

Question:

What is the probability 2 people share the same birthday (month and day)?

The probability is 99.99%!

If there are only 30 people in a room the probability at least 2 people share a birthday is 70.63%!

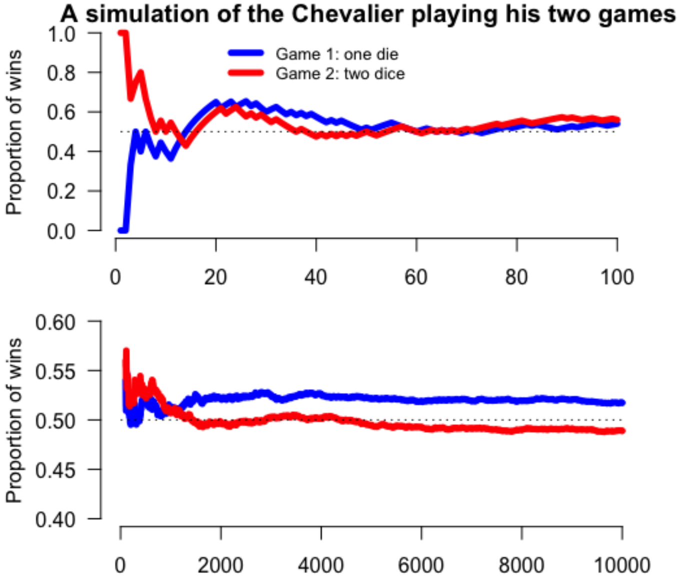
1650's Dice gambling

Chevalier de Méré questioned Pascal which dice game he would be more likely to win:

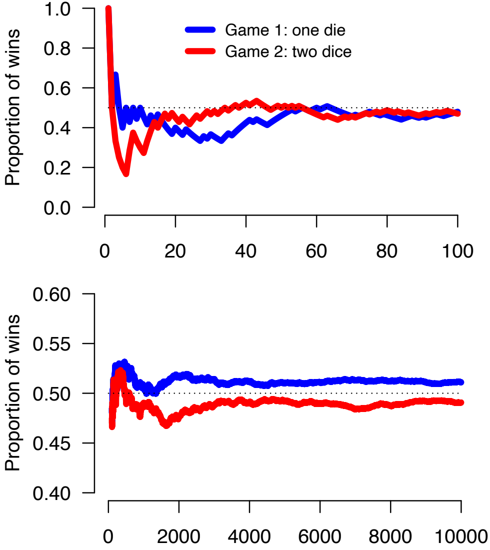
Game 1: Throw one die, up to 4 times, winning once you roll a six.

Game 2: Throw 2 dice, up to 24 times, winning once you roll a double-six.

Short-term v. long-term



Short-term v. long-term: repeated!



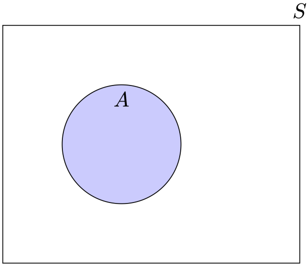
Axioms

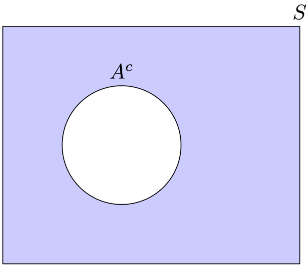
Kolmogorov's Probability Axioms

1. Probabilities cannot be negative
2. Out of all possible outcomes, at least one occuring is 100%
3. Events that are independent (disjoint/mutually exclusive) have a probability equal to adding both event probabilities together.

1. $0 \leq$ P(A)
2. P(S) $= 1$
3. P(A U B) $=$ P(A) $+$ P(B)

Principles: complement



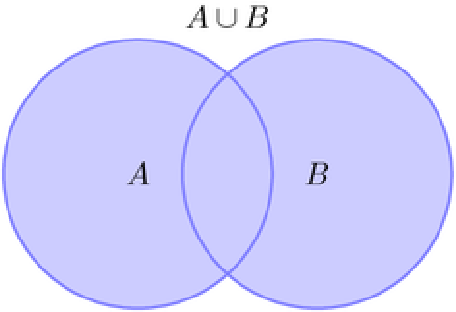


The sample space, S or Ω, is defined as all possible outcomes of a single trial random experiment. On a single dice roll, the outcome can be a value of 1 to 6.

The circle A includes event A: one subset of outcomes. This could be that you roll a 3 on one single dice roll.

A^C is the complement of event A, all other possible outcomes. If you rolled a 3 then the complement is 1, 2, 4, 5, 6.

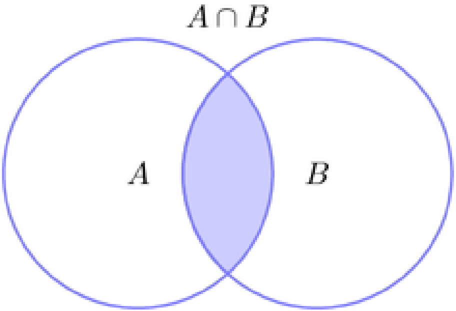
Principles: OR



The event A or B or both A and B happen is the union of A and B.

Set notation is $A \cup B$

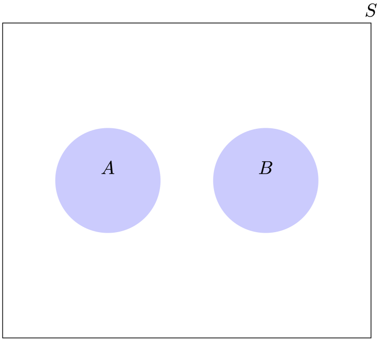
Principles: AND



The event A AND B is the intersection of A and B.

Set notation is $A \cap B$

Principles: disjoint

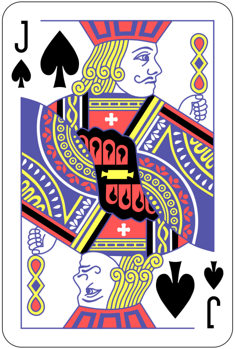


Events A and B do not both happen together is disjoint or mutually exclusive events.

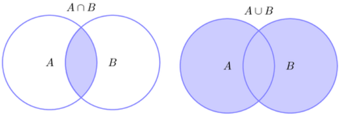
Set notation is $A \cap B = \varnothing$

Union rule

What is the probability of drawing a Jack or a spade?



$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



$$P(Jack \cup \spadesuit) = P(Jack) + P(\spadesuit) - P(Jack \cap \spadesuit)$$

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$$

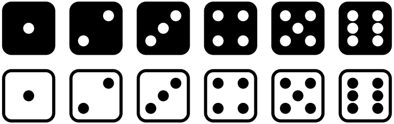
$$= \frac{16}{52} = 30.77\%$$

Independence

Consider independent events as event A will not provide information about event B's occurence.

$$P(A \cap B) = P(A)\ x\ P(B)$$

Rolling dice will not inform you about the next roll.

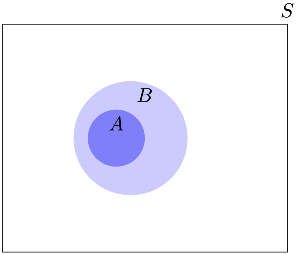


Rolling consecutive sixes on dice rolls is calculated as sample space of 6 events, and the event a 6 occurs twice:

$$\frac{1}{6} x \frac{1}{6} = \frac{1}{36} = 2.78\%$$

Monotonicity

$A \subseteq B = P(A) \leq P(B)$



✓ If you're the president (A) then you are a U.S. citizen (B).
✗ If you're a U.S. citizen then you're president.

Méré's game probabilities

Game 1

P(rolling at least one 6) =
1 - probability(not rolling a 6) =
$1 - \frac{5}{6}\ x\ \frac{5}{6}\ x\ \frac{5}{6}\ x\ \frac{5}{6} = 1 - \left(\frac{5}{6}\right)^4 = 0.5177$

Game 2

6 outcomes x 6 outcomes = 36 total outcomes;
only 1 is double-six;

P(rolling at least one 6) =
1 - (probability not rolling double-6) =
$1 - \left(\frac{35}{36}\right)^{24} = 0.491$

Frequentist: Conditioned on theory

Frequentist

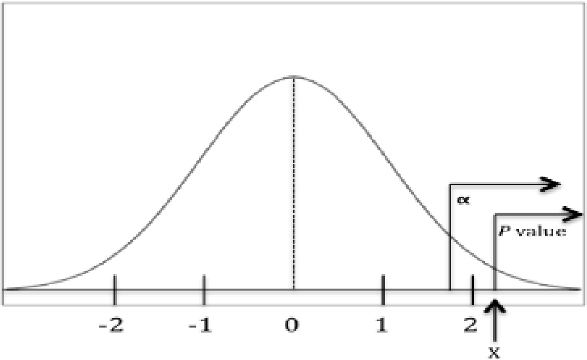
Parameters are fixed and unknown.

Sample data is random from a population distribution.

Neyman-Pearson's Hypothesis test with Fisher's p-value approach

Probability is interpreted as long-run repeated sampling, under almost identical conditions, from a population distribution.

NHST: under the H_0



The p-value is the probability of obtaining a test statistic equal to, or more extreme than, the observed value **under the assumption that the null hypothesis is true.**

Bayesian: Conditioned on data

Bayesian

Parameters are random and unknown.

Sample data is fixed.

Probability is interpreted as our beliefs about random variables; a measure of uncertainty.

Distributions are updated with the arrival of new information—use of priors with the likelihood.

Conditional Probability

Confusing P(A | B), P(B | A), and P(A)...

P(cancer | positive test) is the probability of having cancer when your test results are positive. Is this a false positive or do you have cancer?

P(positive test | cancer) is a tests sensitivity. If you have cancer, what is the chance the test will accurately yield a positive result?

P(cancer) is probability of having cancer; the prevalence.

Bayes theorm

Bayes Theorem

$$P(B|A) = \frac{P(A \mid B) \ x \ P(B)}{P(A)}$$

$$\mathrm{P(B|A)} = \frac{P(A \mid B) \ x \ P(B)}{P(A \mid B) \ x \ P(B) + P(A \mid B^c) \ x \ P(B^c)}$$

Divide a positive result by all the ways it is possible to get a positive result– true positive plus a false alarm.

Example: Breast cancer

The prevalence of breat cancer is 0.8% for women in their forties. The mammogram tests have a 7% false positive rate. The test sensitivity is 90% which means 10% of people with cancer won't be detected by the test.

What is the probability a woman has cancer given a positive test result?

Hint: don't confuse P(cancer|positive result) with P(positive result|cancer)

Example: Breast cancer

P(cancer|positive result)

$$\mathrm{P(BC|t+)} = \frac{\textit{true positive}}{\textit{true positive} + \textit{false alarm}}$$

$$\mathrm{P(BC|t+)} = \frac{(0.9)(.008)}{(0.9)(.008) + (0.07)(.992)}$$

Probability of having cancer given a positive test result $\approx 9.4\%$

X-men: Sentinel project

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Statement: “More mortal people are killed by Sentinels; the estimated numbers in 2019: there were 370 mortal people killed by Sentinels and 235 mutant people killed by the Sentinels”...use probability to check assertion:

Estimated population: 191 million mortal and 42 million mutant.

This means $\dfrac{191}{42} = 4.6$ times more mortal people.

Find expected mutant deaths by Sentinels: $\dfrac{370}{4.6} = 80.43$ mutant people killed by Sentinels if everything was equally proportionate.

Calculate discrepancy: $\dfrac{235}{80.43} = 2.9$.

Which can be interpreted as 3 times more mutant people are killed than mortal people than proportionally expected.

Thank you!

If someone asks "Are you a Bayesian or are you a Frequentist?"
the answer should be yes.