

Process over Product: Math Assessment Tool



“The Right Way To Assess”

The diversity (race, culture, emotional, cognitive, etc) that currently exists in Public School classrooms demands that many traditional standards of student assessments must be abandoned if widespread validation and affirmation of all students is to take place. This level of affirmation and validation can be accomplished without compromising fair and equitable standards of excellence. Simply by entering a classroom, certain demographic groups may be at a distinct academic disadvantage. This academic disadvantage is not correlated to race or culture but is often linked to historic inequities which manifest themselves in poverty.

According to Julia Isaacs (Brookings Institute, 2012), “Poor children in the United States start school at a disadvantage in terms of their early skills, behaviors, and health. Fewer than half (48 percent) of poor children are ready for school at age five, compared to 75 percent of children from families with moderate and high income, a 27 percentage point gap.” Without intervention schools then become factories that simply reproduce poverty.

Teachers must ensure that their classrooms are not factories that perpetuate and reproduce poverty and different classes of people. On the contrary, public schools must be a place where each child is ensured an equitable opportunity to succeed. How students are assessed can go a long way towards cultivating a classroom environment where there is sufficient space for each student (rich and poor) to be affirmed and and to be validated.

Specifically in the mathematics classroom, students are often driven by a need to find a solution or an answer; they are essentially interested in the final product. This is not entirely the fault of the student; teachers and the educational system (obsession with data points) itself are often the drivers of this madness. The fixation on the final product, in my experience, drives students further away from the actual academic prize, which is the process.

The idea behind The Process over The Product Grading system in my mathematics classroom is simple: “Grade the entire process of every assessment item.” On the surface this may seem exhausting, but once you have established your process over the product assessment system the benefits to both student and teacher will be immeasurable.

Teachers must first cherish and then communicate a commitment to process in their lesson planning, instructional activities, and classroom conversations. Students will then begin to value the process themselves and this will become evident in student’s assessment responses. Below is how John, a fictitious student, would fare under a process based assessment system compared to a product based assessment system:

Question 1. Solve the following equation for x: $2x^2 - 12x + 4 = x^2 - 3$	Product Based Assessment For John's Response	Process Based Assessment For John's Response
	$2x^2 - 12x + 4 = x^2 - 3$ Subtract x^2 from both sides $x^2 - 12x + 4 = -3x - 3$ Add $3x$ to both sides $x^2 - 9x + 4 = -3$ Add 3 to both sides $x^2 - 9x + 7 = 0$ Factor to find the zeros $(x - 9)(x + 7) = 0$ Solve $x - 9 = 0 \text{ and } x + 7 = 0$ $x = 9 \text{ and } x = -7$ <i>John is not able to solve for x because he has forgotten how to factor the quadratic equation, so he stops after the first three steps.</i> In a product based system, especially a multiple choice assessment, this response would be graded as incorrect. John's score on this question would be: $\frac{0}{5} \times$	$2x^2 - 12x + 4 = x^2 - 3$ Subtract x^2 from both sides $x^2 - 12x + 4 = -3x - 3$ Add $3x$ to both sides $x^2 - 9x + 4 = -3$ Add 3 to both sides $x^2 - 9x + 7 = 0$ Factor to find the zeros $(x - 9)(x + 7) = 0$ Solve $x - 9 = 0 \text{ and } x + 7 = 0$ $x = 9 \text{ and } x = -7$ The teacher writes: "John, Good work on simplifying the equation. Think about the following: "what are the factors of 8 when you multiply those factors you will get 8, but when you add them you will get -9. Good job, thus far, see me after class so we can discuss." In the process based system, John receives the following score: $\frac{3.5}{5}$

Think About It

If this was a multiple choice exam with 10 similar questions, John would likely score a 0/50 (0% or an F) using the product based assessment system. Conversely, in the process based assessment system, John would likely score a 35/50 (70% or a C). John is aware that he did a portion of the questions correctly. Validating and affirming him for his work would lift his self esteem and position him emotionally to better comprehend factoring. There are many Johns in our classroom.

Here is the justification, in granular detail, for John's score of 3.5/5 on this particular assessment item

- John recognizes that this is an equation and not an expression
- He recognizes and knows what like terms are what he can do to combine them
- John inherently knows that he can only combine variables that are raised to the same power.
- He knows that when he combines these like variables he is only adding their coefficients.
- John knows that he can combine constant terms.
- He understands the rules of equality.
- He understands that if he adds a term on the left that he must also add a term on the right; the addition property of equality. (John adds $3x$ on the right and then adds $3x$ on the left; he also adds 4 on the right and then adds 4 on the left..)
- John understands that if he subtracts a term on the right he must also subtract that same term from the left; the subtraction property of equality. (John subtracts x^2 from the right and also subtracts x^2 from the left.
- John's process is very efficient because he ensures that the leading term, x^2 , remains positive.
- Although John has factored the quadratic expression on the left incorrectly, he knows that he must have two factors $(x-9)(x+8) = 0$
- By setting the factors equal to zero and then solving can imply that John understands that the x values are solutions of this quadratic equation.
- It can also be inferred that he understands that these x values are also x -intercepts.
- John correctly sets these incorrect factors to zero and then solves them.
- John knows a whole lot about this problem. The only thing in John's process that is entirely incorrect is the final result; or the product of John's work.
- After considering the amount of math that John actually knows relative to this problem, should his score be elevated above a 3.5/5?

$$2x^2 - 12x + 4 = x^2 - 3x - 4$$

Subtract x^2 from both sides

$$x^2 - 12x + 4 = -3x - 4 \checkmark$$

Add $3x$ to both sides

$$x^2 - 9x + 4 = -4 \checkmark$$

Add 4 to both sides

$$x^2 - 9x + 8 = 0 \checkmark$$

Factor to find the zeros

$$(x - 9)(x + 8) = 0 \quad \left(\frac{1}{2}\checkmark\right)$$

Tips for a successful Process Based Assessment System

- Get to know your student
- Get to know your content area intimately
- Reflect on individual student academic needs as you lesson plan
- Ensure that math lessons show a connection between math topics (students are likely to remember previous content when it is connected to current content).
- Constantly connect the smaller ideas of math to the bigger ideas of math and then connect the bigger ideas to the student's real life and to the world
- Construct assessments as you lesson and unit plan
- Ensure that assessment items are valid by deriving them from the lesson or unit standards and objectives
- Ensure that the lesson and unit objectives are clearly met
- Review assessment data, over time and over periods, to ensure that assessment items are reliable
- Discuss all assessment items with students, especially outlier items, post test. This will provide insight for future test construction
- Create your own assessment items; this increases knowledge of the content and intimacy with the content.
- Explode each assessment item prior to the assessment to ensure that you know all of the steps (process) and permutations to a possible solution.
- Be aware that a student can also present a process and a solution that you did not consider
- Assign each step of the solution process a fraction of the item's total score.
- Grade the entire process and not only the solution
- Assess orally during classroom discussions and create a culture of process over product
- Openly encourage different paths to solutions

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An example of a Teacher made exam with two exploded process based solutions

Algebra 2

Logarithms Exam

Name:

Please be explicit in all answers that you provide. The purpose of this exam is to determine your ability to apply the properties that are related to logarithms. Questions without proper explanations will be marked as incorrect. All final logarithmic expressions, prior to using a calculator, must be in base 10 (common log) or base e (ln). All submitted work must be organized, numbered and legible. The teacher reserves the right to verify by oral examination that the responses provided were entirely generated by you. If an answer that you generate is not a valid solution, please indicate why. The exam will be 60 minutes long. Students with extra time, designated as such by the State of Florida, will have the entire period.

When you have completed the exam, your responses must be uploaded to TEAMS.

<p>Simplify.</p> <p>1.) $\log_{\frac{4}{5}}(4.102)^5$</p> <p>$5\log_{\frac{4}{5}}(4.102)$ application of the power rule (✓ 2pts)</p> <p>$5\left(\frac{\log(4.102)}{\log(\frac{4}{5})}\right)$ application of change of base (✓ 2pts)</p> <p>$5\left(\frac{0.613}{-0.097}\right) = -6.319$ input into calculator (✓ 1pt)</p> <p>5 possible points</p>	<p>2.) Simplify using properties of logarithms and make sure that there are no fractions or negative exponents remaining in the arguments.</p> <p>$\log_{\frac{1}{x^2}}(4.102)^4$</p>
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<p>3.) Use properties of logarithms to solve the following exponential equation.</p> $\log_{15}(3375)^{(x+3)} = \log_{15}\left(\frac{1}{225}\right)^{-6x+22}$ <p>$(x + 3)\log_{15}(3375) = (-6x + 22)\log_{15}\left(\frac{1}{225}\right)$ power rule ✓ 2pts</p> <p>$(x + 3)\frac{\log(3375)}{\log(15)} = (-6x + 22)\log_{15}(15^{-2})$ change and negative exponent rule ✓ 2pts</p> <p>$(x + 3)\frac{\log(3375)}{\log(15)} = (12x - 44)\log_{15}(15)$ power rule, and the distributive property ✓ 2pts</p> <p>$(x + 3)\frac{\log(3375)}{\log(15)} = (12x - 44)(1)$ definition based of logarithms: $\log_b b = 1$ ✓ 1pts</p> <p>$(x + 3)(3) = (12x - 44)(1)$ calculator use ✓ 1pts</p> <p>$3x + 9 = 12x - 44$ distributive property ✓ 1pts</p> <p>$53 = 9x$ subtraction and addition property of equality ✓ 2pts</p> <p>$x = \frac{53}{9}$ solution ✓ 1pts</p> <p>12 possible points</p>	<p>4.) Use properties of logarithms to solve the following exponential equation.</p> $\log_3(27)^{(x+3)} = \log_4(64)^{-2x+4}$
<p>5.) Simplify. Please justify each step. Use properties of logarithms to solve the following exponential equation.</p> $\log_{.37x}(.37x)^{1/5} = \log_{.002y}(.002y)^{1/5}$	<p>6.) If $f(x) = \log_4 x$ What is $F(4096)$. Please with an exponential equivalent.</p>
<p>7.) If $f(x) = \log_{\frac{1}{7}} x$ What is $F(2401)$. Please confirm with an exponential equivalent.</p>	<p>8.) Simplify the following expression and show all work:</p> $\log_{\frac{1}{11}}(1331) + \log_{10}(1000)^{3x}$
<p>9.) Simplify the following expression and show all work:</p> $\log_9\left(\frac{1}{729}\right)^x - \log_{14}(2744)^x$	<p>10.) Solve for x using the properties of logarithms. Please show all of your work:</p> $\ln(x) + \ln(x^3) = 81$
<p>11.) Solve for x using the properties of logarithms:</p> $\log_{\frac{3}{x}}(11x + 2) = \log_{\frac{3}{x}}(x^2 + 30)$ $\left(\frac{27}{x^3}\right)^{\frac{1}{3}}$	<p>12.) Solve for x:</p> $\log_{17}(-22x) - \log_{17}(x^2 + 30) = \log_{17}(2)$

<p>13.) Completely explain the transformation of the following logarithmic function as compared to the parent function ($\log_3 x$).</p> $f(x) = 12 \log_3(x - 21) + 12$	<p>14.) For question 13, determine the x and the y intercepts and the location of the vertical asymptote if there is one. Show all work.</p>
<p>15.) The population of bacteria in a culture after t minutes is given by the equation $P = P_o (1.12)^t$, where P_o is the initial population. If the number of bacteria starts at 12,000, how long will it take, in minutes and seconds, to get to 145,000.</p>	<p>16.) Bonus Question: Consider the following logarithmic function and completely determine its transformation as it relates to its parent function.</p> $f(x) = (-1.75) \log_{\frac{1}{8}}(x - 0.25) + 1.8$ <p>Identify the following:</p> <ul style="list-style-type: none"> • The parent function of the given function: $f(x) = \log_{\frac{1}{8}}(x)$ • The x-intercept of the given function • The y-intercept of the given function • The location of the asymptote • Determine if the function is compressed or stretched and by what factor. • Determine if the function is reflected. • The domain of the function • The range of the function

In a process based assessment system, students who are traditionally marginalized and negatively labeled in a traditional classroom will have an opportunity for endless affirmations and validations.

Citation:

Isaacs, J. (2012, March 19). Starting School at a Disadvantage: The School of Readiness for Poor Children.

<https://www.brookings.edu/research/starting-school-at-a-disadvantage-the-school-readiness-of-poor-children/>